Self-Referential Probability

I study languages that can express:

(\pi) The probability of \pi is not greater than or equal to \frac{1}{2}.

Connection to the liar paradox:

(\lambda) \lambda is not true

Seemingly harmless principles like introspection now lead to contradictions.

Why consider such sentences? §1.1.1

These arise in languages that can express:

• Georgie is (probabilistically) certain that Dan believes to degree \frac{1}{2} that the coin will land heads.
• Every sentence has probability greater than or equal to 0.

Can give us options for how to deal with cases like promotion:

Alice will get a promotion just if she does not have a degree of belief greater than or equal to \frac{1}{2} that she’ll get it.

Developing a semantics Part I

Use possible world structures to provide the facts about probability. E.g.

\begin{center}
\begin{tikzpicture}
\node at (0,0) {Heads \ / \ / \ Tails} node at (1,0) {1/2} node at (2,0) {1/2} node at (3,0) {1/2} ;
\end{tikzpicture}
\end{center}

Allows for varying extensions of P.

• The obvious definition

\[ w \models_{\mathfrak{M}} \frac{p}{\geq r} \varphi \iff m(w \cdot v \models \varphi) \geq r, \]

is often not satisfiable. (ch. 2)

Useful technique: Generalise semantics given for the liar paradox.

• Kripkean semantics – Non-classical probabilities.
  – Strong Kleene (ch. 3) – Also obtain an axiomatisation.
  – Supervaluational (ch. 4) – Provides imprecise probabilities.
• Revision theory (ch. 5) – Classical probabilities, but non-terminating sequence of models.
Rationality Requirements Part II

The accuracy argument (ch. 7) and the Dutch book argument (ch. 8).
I formalise and study proposals by Caie:

Consider how good a credal state would be were the agent to adopt it (i.e. if it were the interpretation of $P$).

- Leads to unwieldy rationality constraints (§7.2)
  - Non-probabilistic,
  - Non-introspective,
  - Negative,
  - Non-logically-omniscient.

We should instead:

Evaluate a credal state from the initial credal state’s perspective.

Different formulations of this allow for the different semantics as developed in part I.