Rational Probabilistic Incoherence? A Reply to Michael Caie*

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Abstract

In Michael Caie's paper 'Rational Probabilistic Incoherence', Caie argues that in light of certain situations involving self-reference it is sometimes rational to have probabilistically incoherent credences. In this paper we further consider his arguments. We show that probabilism isn't at fault for the failure of rational introspection and that Caie's modified accuracy criterion conflicts with Dutch book considerations, is scoring rule dependent and leads to the failure of rational introspection.

In Michael Caie's paper 'Rational Probabilistic Incoherence' [Caie, 2013], Caie argues that it is sometimes rational to have probabilistically incoherent credences and therefore that probabilism is false. Our paper provides further analysis of his arguments and the position he ends up in.

Caie first observes that if probabilism holds then an agent is rationally required to have poor epistemic access to some of her own credences. He presents this as a prima facie problem for probabilism. In Section 1 we shall show that assumptions weaker than probabilism lead to the requirement of poor epistemic access and, given one way of understanding epistemic access, only very basic requirements on credences are needed for the result. This all suggests that probabilism is not to blame for the failure of rational introspection.

Caie admits that this conflict between probabilism and rational introspection is merely a *prima facie* problem for probabilism. He says:

> We might simply accept this consequence of probabilism despite its prima facie implausibility. In this essay, I'll argue that this isn't the right response. *To this end* I'll show that [a case used in the prima facie problem for probabilism]. . . can be used to expose flaws

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in the accuracy-dominance argument for probabilism. Once these flaws are exposed, we can see that considerations of accuracy, instead of motivating probabilism support the claim that a rational agent's credences should be probabilistically incoherent. ([Caie, 2013, pg. 528]; our emphasis)

One might therefore suppose that Caie's proposed modified accuracy dominance criterion would allow an agent to have good epistemic access to her own credences. But we show in Section 2 that this is not the case. This observation doesn't weaken his argument for rational probabilistic incoherence but instead throws light on the connection between Caie's two arguments.

In Sections 3 and 4 we point out some previously unnoticed features of Caie's proposed accuracy criterion. In Section 3 we show that the credences which are rationally required according to Caie's proposed accuracy criterion lead to sure losses on some bets and that the agent's sure losses could be minimized by holding other credences. This is an undesirable feature and shows that Caie cannot use Dutch book considerations to support his criterion. Caie shows in his paper that Dutch book considerations in fact don't support probabilism so if Caie's modification of the accuracy criterion is rejected we still do not avoid the conflict between accuracy and Dutch book considerations. Instead we should see this as an undesirable feature of accuracy considerations that has to be dealt with if accuracy considerations are to be accepted.

In Section 4 we show that the criterion is dependent on the scoring rule on which it is based. Although different choices of scoring rules all still lead to rational probabilistic incoherence, so this again doesn't weaken Caie's argument for rational probabilistic incoherence, they lead to different credal states being deemed rational. Therefore one needs to do more work to explain what rational constraints accuracy domination considerations lead to if Caie's modification is accepted. At the end of Section 4 we present some options for answers to this question.

1 A problem for Caie's prima facie argument against probabilism

The principle that Caie argues against is:

Probabilism: *An agent is rationally required to have probabilistically coherent credences.*

where an agent's credences, Cr, are probabilistically coherent if they satisfy:[1]

- For any necessary proposition,[2] $\top$, $\mathsf{Cr}(\top) = 1$

[1] As Caie does, we understand Cr as a function from propositions to real numbers and we present a propositional version of the axioms of probability.

[2] Caie originally stated this condition as "For any logical truth". For Caie's arguments to work he needs Propositions to hold so that for example $\mathsf{Cr}\rho(\#) = \mathsf{Cr}\rho(`\mathsf{Cr}\rho(\#) \geqslant 0.5')$. This follows from the axioms if we require for example that the proposition $\rho(\#) \leftrightarrow \rho(`\mathsf{Cr}\rho(\#) \geqslant 0.5')$ has probability 1 although it is not a *logical* truth.

- For any proposition, φ, $0 \leqslant \mathsf{Cr}(\varphi)$
- If φ and ψ are incompatible propositions, i.e. it is necessary that $\neg(\varphi \wedge \psi)$, then $\mathsf{Cr}(\varphi \vee \psi) = \mathsf{Cr}(\varphi) + \mathsf{Cr}(\psi)$

The two arguments which Caie gives for rejecting Probabilism both rely on a sentence whose truth depends on an agent's credences. This sentence is #, which is introduced by stipulating that the name '#' refers to the following, interpreted, sentence:

> Hiro's credence in the proposition expressed by # isn't greater than or equal to 0.5.

He uses 'Cr_H' to abbreviate 'Hiro's credence in' and 'ρ' to abbreviate 'the proposition expressed by'.[3] The above can then be represented by:

$$(\#) \qquad \neg\, \mathsf{Cr}_H \rho(\#) \geqslant 0.5$$

In this paper we take for granted the existence of such propositions. [4]

The first argument which Caie gives against probabilism relies on the following theorem.

Theorem 1. *If Hiro's credences are probabilistically coherent then, given the existence of #, one of the following must hold:*

Neg Insensitivity: $\neg\, \mathsf{Cr}_H \rho(\#) \geqslant 0.5$ *and* $\neg\, \mathsf{Cr}_H \rho(\text{`}\neg\, \mathsf{Cr}_H \rho(\#) \geqslant 0.5\text{'}) > 0.5$

Pos Insensitivity: $\mathsf{Cr}_H \rho(\#) \geqslant 0.5$ *and* $\neg\, \mathsf{Cr}_H \rho(\text{`}\mathsf{Cr}_H \rho(\#) \geqslant 0.5\text{'}) > 0.5$

Proof. One consequence of having probabilistically coherent credences is:

Propositions: *Credences are assigned to propositions.*

By assumption $\rho(\#) = \rho(\text{`}\neg\, \mathsf{Cr}_H \rho(\#) \geqslant 0.5\text{'})$, so using Propositions, $\mathsf{Cr}_H \rho(\#) = \mathsf{Cr}_H \rho(\text{`}\neg\, \mathsf{Cr}_H \rho(\#) \geqslant 0.5\text{'})$. Using this we can see that either $\neg\, \mathsf{Cr}_H \rho(\#) \geqslant 0.5$ and $\neg\, \mathsf{Cr}_H \rho(\text{`}\neg\, \mathsf{Cr}_H \rho(\#) \geqslant 0.5\text{'}) \geqslant 0.5$ holds, or $\mathsf{Cr}_H \rho(\#) \geqslant 0.5$ and $\mathsf{Cr}_H \rho(\text{`}\neg\, \mathsf{Cr}_H \rho(\#) \geqslant 0.5\text{'}) \geqslant 0.5$ holds. Neg Insensitivity immediately follows from the former. Pos Insensitivity follows from the latter using another consequence of Cr being probabilistically coherent, which we call Negation.

Negation: $\mathsf{Cr}\rho(\text{`}\neg A\text{'}) \geqslant 0.5 \implies \neg\, \mathsf{Cr}\rho(\text{`}A\text{'}) > 0.5$

[3] We use 'Cr' as an abbreviation for 'Hiro's credence in', so the function from propositions to reals which it denotes depends on the situation. a, b, c and d are used to rigidly denote functions from propositions to reals. We use Cr as the variable which might be instantiated by such a, b etc.

[4] One could instead consider Hiro assigning credences to sentences of a language that contains predicates like '$\mathsf{Cr}_H(\cdot) \geqslant 0.5$' that apply to codes of sentences, taking a background theory of arithmetic coding the sentences. In that theory one could therefore derive the diagonal lemma and result in a sentence # where $\# \leftrightarrow \neg \mathsf{Cr}_H \ulcorner \# \urcorner \geqslant 0.5$ is a theorem. Assuming the background arithmetic theory is taken to be necessary, Probabilism will still imply Propositions, where the proposition $\rho(\text{`}A\text{'})$ is taken to be the set of models (of the background theory) satisfying A. We stick to Caie's example of Hiro instead of Yuko because it stays closer to this way of understanding things.

□

Therefore if Probabilism holds then Hiro is rationally required to satisfy either Neg Insensitivity or Pos Insensitivity.

Caie says "it is prima facie implausible that an agent could be rationally required to have poor epistemic access to her own credal state"[Caie, 2013, pg. 4]. He therefore supports the following principle:

Weak Rational Introspection: *An agent should not be rationally required to have poor epistemic access to her own credal state.*

If either Neg Insensitivity or Pos Insensitivity hold then Hiro is insensitive to his own credences. So to maintain Weak Rational Introspection, Probabilism must be rejected.

The only components of Probabilism required for this proof were that Propositions and Negation are rationally requirements, which is much more innocent than Probabilism.[5] It would be particularly difficult to reject Propositions since it is a basic assumption about credences. In fact Caie supports Propositions throughout his paper. Instead he says a rational agent may fail to satisfy Negation and claims there are situations where a rational agent should have credence $\mathsf{Cr}\rho(`\neg A') \geqslant 0.5$ whilst still holding that $\mathsf{Cr}\rho(`A') > 0.5$. This is itself prima facie implausible, perhaps more so than the requirement to have poor epistemic access to one's own credal states.

Even rejecting the rational requirement of Negation does not obviously allow one to avoid the requirement to have poor epistemic access to ones own credal state. This is because in the proof of Theorem 1 we only used Negation to derive Pos Insensitivity from

Alternative Pos Insensitivity: $\mathsf{Cr}_H\rho(\#) \geqslant 0.5$ *and* $\mathsf{Cr}_H\rho(`\neg\, \mathsf{Cr}_H\rho(\#) \geqslant 0.5') \geqslant 0.5$

So by just using Propositions we could already conclude that if Hiro is rational then either Neg Insensitivity or Alternative Pos Insensitivity hold. If Hiro satisfies Alternative Pos Insensitivity then although he is in some credal state, $\mathsf{Cr}_H\rho(\#) \geqslant 0.5$, he has high credence that he is not in that credal state. One might therefore take Alternative Pos Insensitivity to be a case of Hiro being insensitive to his own credences. Under that assumption the rational requirement to satisfy Propositions conflicts with Weak Rational Introspection and therefore Weak Rational Introspection would have to be rejected since it is much more plausible that a rational agent must satisfy Propositions.

One might instead reject that Alternative Pos Insensitivity is an instance of Hiro being insensitive to his own credences, perhaps by arguing that our intuition that Alternative Pos Insensitivity displays insensitivity relies on an implicit acceptance of Negation. However even if Caie was to do this, Weak Rational Introspection will still be problematic for him. This is because the way that Caie argues that accuracy considerations should apply will lead to the failure of Weak Rational Introspection.

[5] For example a Dempster-Shafer belief function will satisfy Negation and Propositions.

2 Accuracy Domination Arguments Lead to the Failure of **Weak Rational Introspection**

The argument against probabilism that Caie most strongly supports is that accuracy-dominance considerations in fact lead to rational probabilistic incoherence. We now show that his accuracy dominance criterion conflicts with Weak Rational Introspection.

The accuracy argument assumes the goal of an agent is to have credences that are as accurate as possible. Caie argues that we should only consider how accurate a credal state would be *if they were the agent's credences.* So for propositions whose truth value is determined by an agent's credences, such as $\rho(\#)$, the only relevant accuracy score is the accuracy of the credal state at the world where the agent has that credal state. Since all our examples are based on cases where the credal state is only defined over such propositions we just state the criterion for these cases.

Caie focuses on the Brier score, BS, as a measure of the accuracy of an agent's credal states at a world and we shall initially do the same.

Caie's Accuracy Criterion: *Let $\varphi_1 \dots \varphi_n$ be a finite algebra*[6] *of distinct propositions whose truth depends only on what the agent's credences are. Let* Cr *be some credal state, i.e. an assignment of real numbers to the propositions $\varphi_1, \dots, \varphi_n$. Define*

$$\mathsf{U}(\mathsf{Cr}) := 1 - \mathsf{BS}(w_{\mathsf{Cr}}, \mathsf{Cr}) = 1 - \frac{1}{n}\sum_{i=1}^{n}(w_{\mathsf{Cr}}(\varphi_i) - \mathsf{Cr}(\varphi_i))^2$$

where w_{Cr} is the world that would be actual if Cr *were the agent's credences.*

If credal state Cr *is such that for all other credal states* Cr', $\mathsf{U}(\mathsf{Cr}) > \mathsf{U}(\mathsf{Cr}')$, *then an agent is rationally required to have credal state* Cr.

In Hiro's case this criterion leads to $\mathsf{Cr}\rho(\#) = 0.5$ and $\mathsf{Cr}\rho(\neg\#) = 1$ as the rationally required credal state. Since this is a probabilistically incoherent credal state Caie concludes that accuracy considerations lead to the rejection of Probabilism.

We can now present our example that shows that Caie's proposal conflicts with Weak Rational Introspection.

Instead of considering # we consider γ which is a minor modification of #. Let 'γ' name the following sentence:

> June's credence in the proposition expressed by γ isn't greater than 0.5.

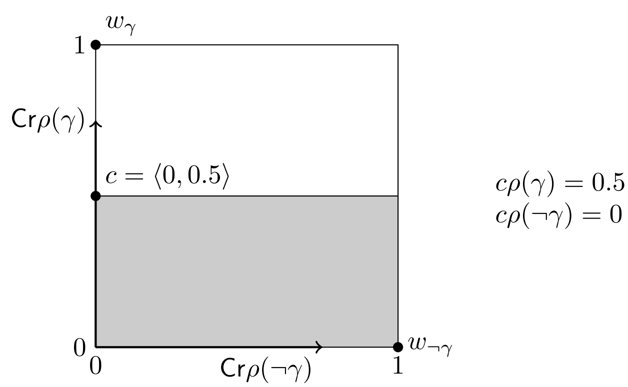
This can be represented by:

$$(\gamma) \qquad \neg\, \mathsf{Cr}_J\rho(\gamma) > 0.5$$

We can represent this situation diagrammatically as Caie does in his paper. Since the only propositions we are interested in are $\rho(\gamma)$ and $\rho(\neg\gamma)$ we shall just

[6] Namely a collection of propositions which is closed under negations and finite disjunctions. We ignore $\top$ and $\bot$ and will assume that these will always be assigned credences 1 and 0 respectively.

consider June's credences in these two propositions. We can therefore represent June's possible credences by points on the graph whose axes are $\mathsf{Cr}\rho(\neg\gamma)$ and $\mathsf{Cr}\rho(\gamma)$. The two possible states of affairs are w_γ, where γ is true, and $w_{\neg\gamma}$, where γ is false. As Caie does we shall also identify these as points in the diagram, as labelled below, and we shall only focus on the credal states in $[0,1]^2$. Unlike Caie we also shade the credal states where if they were June's credences then γ would be true.



Theorem 2. *According to* Caie's Accuracy Criterion *June is rationally required to be in state c.*

Proof.

$$\mathsf{U}(\mathsf{Cr}) = \begin{cases} 1 - \mathsf{BS}(w_{\neg\gamma}, \mathsf{Cr}) & \text{if } \mathsf{Cr}\rho(\gamma) > 0.5 \\ 1 - \mathsf{BS}(w_\gamma, \mathsf{Cr}) & \text{otherwise} \end{cases}$$

$$\mathsf{Cr}\rho(\gamma) > 0.5 \implies \mathsf{BS}(w_{\neg\gamma}, \mathsf{Cr}) = \frac{\mathsf{Cr}\rho(\gamma)^2 + (1 - \mathsf{Cr}\rho(\neg\gamma))^2}{2} > \frac{0.5^2}{2} = 0.125$$

$$\neg\, \mathsf{Cr}\rho(\gamma) > 0.5 \implies \mathsf{BS}(w_\gamma, \mathsf{Cr}) = \frac{(1 - \mathsf{Cr}\rho(\gamma))^2 + \mathsf{Cr}\rho(\neg\gamma)^2}{2}$$

$$\geqslant \frac{(1-0.5)^2}{2} = 0.125$$

So the minimal $\mathsf{U}(\mathsf{Cr})$ possible is 0.125. One can check that this is obtained only at c. □

Credal state c has the properties $\neg\ c\rho(\gamma) > 0.5$ and $\neg\ c\rho($'$\neg\ \mathsf{Cr}_J\rho(\gamma) > 0.5$'$) > 0.5$. This is directly analogous to Neg Insensitivity as we stated it above.[7] Therefore c is a credal state where June has poor epistemic access to her own credences. According to Caie's accuracy criterion c is a rationally

[7] We might write Neg Insensitivity more generally as: June has Neg Insensitivity if there is some proposition φ and interval Δ such that $\neg\ \mathsf{Cr}_J\varphi \in \Delta$ and $\neg\ \mathsf{Cr}_J\rho($'$\neg\ \mathsf{Cr}_J\varphi \in \Delta$'$) > 0.5$. c has Neg Insensitivity in this sense.

required credal state. This shows that Caie's Accuracy Criterion is inconsistent with Weak Rational Introspection even if Alternative Pos Insensitivity is not taken to be an instance of having poor epistemic access to one's own credences. One can deal with this inconsistency either by rejecting Caie's Accuracy Criterion or Weak Rational Introspection, though it is important to note that this does not weaken an argument against Probabilism since each of Caie's Accuracy Criterion and Weak Rational Introspection lead to rejecting Probabilism. The inconsistency instead shows that accuracy considerations do not support the prima facie plausible principle of Weak Rational Introspection even if Caie's modification of the accuracy considerations is accepted.

In the next two sections we will present some features of Caie's Accuracy Criterion which were previously unnoticed.

3 Caie's Accuracy Criterion leads to needless loss

As Caie mentions in his paper, due to sentences like # sometimes an agent will always value as fair a set of bets which guarantee a loss of money. He says that in such situations Dutch book considerations should require an agent to minimize her loss. However, we show here that there are some situations where Caie's Accuracy Criterion leads to an agent being rationally required to hold credal states that lead to needless loss. This shows that Caie cannot use Dutch book considerations to support his criterion.

Caie says that Dutch book considerations should lead one to accept Loss Minimization:

Loss Minimization: *If possible, an agent is rationally required to have a credal state which, assuming she bets with her credences, minimizes her possible losses.*[8]

We shall show that Caie's Accuracy Criterion is incompatible with a weaker principle, namely Weak Loss Minimization.

Weak Loss Minimization: *It is rationally permissible for an agent to have a credal state that minimizes her possible loss, assuming she bets with her credences, if there is some such credal state.*

[8] We understand this as that the agent should minimize her *average* possible losses. In the case that Caie considered there is a single credal state which minimizes the agent's losses on each bet which guarantees a loss (or break-even), but there is no such credal state in the case that we consider, so this is how we suggest the criterion should be applied in our case. As Caie does we only consider bets instead of sets of bets. We also only consider unit bets where there is $1 at stake (we do not loose any information by doing this because the loss on a bet which offers $\$s$ is s-times the loss on the equivalent bet offering $1). More carefully, we say the agent minimizes her possible losses when she minimizes

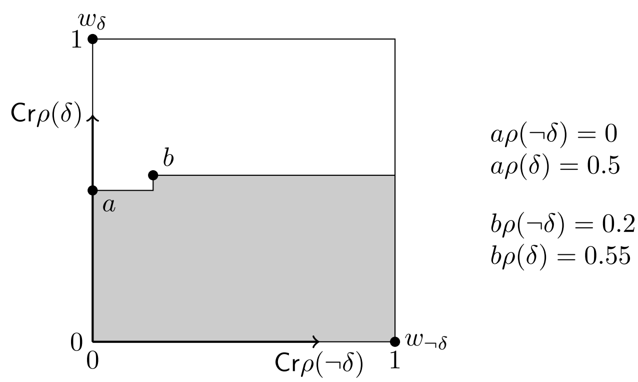
$$\frac{\mathsf{Loss}(\varphi_1) + \ldots + \mathsf{Loss}(\varphi_n)}{n}$$

where $\mathsf{Loss}(\varphi)$ is the loss on whichever of [$1 if φ] and [-$1 if φ] leads to a loss and 0 if they both lead to a break-even (this is a good definition because the gain on [$1 if φ] is minus the gain on [-$1 if φ]). It is interesting to observe that a also minimizes the average loss when sets of bets are considered, under particular ways of measuring this.

Consider the following, admittedly unusual, sentence about Roy's credences:[9]

$$(\delta) \qquad \mathsf{Cr}_R\rho(\delta)\leqslant 0.5 \vee (\mathsf{Cr}_R\rho(\delta)\leqslant 0.55 \wedge \mathsf{Cr}_R\rho(\neg\delta)\geqslant 0.2)$$

As before, we can represent this situation diagrammatically.



Theorem 3. *According to* Caie's Accuracy Criterion *Roy is rationally required to be in credal state b, which is a credal state that, assuming Roy bets with his credences, will not minimizes his possible losses (whereas a does minimize them).*[10] *Therefore* Caie's Accuracy Criterion *and* Weak Loss Minimization *are incompatible principles.*

Proof. Consider the credal states a and b and the unit bets on δ, namely:

[\$1 if δ] and [-\$1 if δ].

At both a and b it is the second one which will lead to a loss because if the agent is in credal state a or b, δ is true. The loss on this bet is:

a: $1 - 0.5 = \$0.5$
b: $1 - 0.55 = \$0.45$

For the bets on $\neg\delta$ it is the positive one which will lead to a loss at b and both the positive and negative lead to a break-even at a. The losses Roy can be forced into by taking bets on $\neg\delta$ are therefore:

a: \$0
b: \$0.2

[9] We could instead consider a proposition φ where Roy's credence in $\varphi \leftrightarrow \mathsf{Cr}_R\varphi \leqslant 0.5 \vee (\mathsf{Cr}_R\varphi\leqslant 0.55 \wedge \mathsf{Cr}_R\neg\varphi\geqslant 0.2)$ is high instead of requiring that this holds of necessity. Then we could modify our results to see that the credences that are expectedly most accurate do not minimize Roy's expected average possible loss, and the expectedly most accurate credal state differs depending on whether the Brier score or logarithmic score are used. For simplicity we shall not do this.

[10] In the sense of Footnote 8.

So Roy's average (also total) possible losses will be minimized at a. One can check that a in fact minimizes his possible losses.

We now show that b is rationally required according to Caie's Accuracy Criterion. We only give the argument explicitly for a and b, since these are the states which are competing for being the most accurate.

$$\mathsf{U}(\mathsf{Cr}) = \begin{cases} 1 - \mathsf{BS}(w_\delta, \mathsf{Cr}) & \text{if } \mathsf{Cr}\rho(\delta) \leqslant 0.5 \vee (\mathsf{Cr}\rho(\delta) \leqslant 0.55 \wedge \mathsf{Cr}\rho(\neg\delta) \geqslant 0.2) \\ 1 - \mathsf{BS}(w_{\neg\delta}, \mathsf{Cr}) & \text{otherwise} \end{cases}$$

$$\mathsf{BS}(w_\delta, a) = \frac{0^2 + (1-0.5)^2}{2} = 0.125$$
$$\mathsf{BS}(w_\delta, b) = \frac{0.2^2 + (1-0.55)^2}{2} \approx 0.121$$

Therefore $\mathsf{U}(a) < \mathsf{U}(b)$. By checking that all other credal states Cr have $\mathsf{U}(\mathsf{Cr}) < \mathsf{U}(b)$ we can see that credal state b is rationally required according to Caie's Accuracy Criterion. □

This shows that Caie's Accuracy Criterion is not compatible with Weak Loss Minimization. Although this is an undesirable feature of Caie's Accuracy Criterion the unmodified accuracy criterion has the same undesirable feature.[11] It is nonetheless interesting that accuracy considerations and Dutch book considerations conflict and it is a consequence of Caie's Accuracy Criterion which one should be aware of.

4 Scoring Rule Dependence of Caie's Accuracy Criterion

Not only does Caie's Accuracy Criterion conflict with Dutch book considerations, it also conflicts with criteria analogous to Caie's Accuracy Criterion which are based on different scoring rules. This is in contrast to the usual formulation of the accuracy criterion.

We can use the above example to show that the criterion leads to different rational requirements depending on whether the Brier score or the logarithmic score are used.

Theorem 4. *Roy is rationally required to be in credal state a, according to the criterion analogous to* Caie's Accuracy Criterion *but based on the logarithmic scoring rule.*[12]

[11] Which can be seen because Weak Loss Minimization leads to rational probabilistic incoherence whereas the usual accuracy criterion does not.

[12] This is given by $\mathsf{LS}(w, \mathsf{Cr}) = \frac{1}{n}\sum_{i=1}^{n} -\ln(|(1-w(\varphi_i)) - \mathsf{Cr}(\varphi_i)|)$. One might think that the accuracy criterion based on the logarithmic scoring could be compatible with Weak Loss Minimization, but this is also not the case. To see this consider

$$(\chi) \qquad \mathsf{Cr}_D\rho(\chi) \leqslant 0.5 \vee (\mathsf{Cr}_D\rho(\chi) \leqslant 0.7 \wedge \mathsf{Cr}_D\rho(\neg\chi) \geqslant 0.25)$$

Proof.

$$\mathsf{U}_{\mathsf{LS}}(\mathsf{Cr}) = \begin{cases} 1 - \mathsf{LS}(w_{\neg\delta}, \mathsf{Cr}) & \text{if } \mathsf{Cr}\rho(\delta) \leqslant 0.5 \vee (\mathsf{Cr}\rho(\delta) \leqslant 0.55 \wedge \mathsf{Cr}\rho(\neg\delta) \geqslant 0.2) \\ 1 - \mathsf{LS}(w_{\delta}, \mathsf{Cr}) & \text{otherwise} \end{cases}$$

We only need to explicitly consider a and b since it is clear that any other $\mathsf{U}_{\mathsf{LS}}(\mathsf{Cr})$ will be lower than either $\mathsf{U}_{\mathsf{LS}}(a)$ or $\mathsf{U}_{\mathsf{LS}}(b)$.

$$\mathsf{U}_{\mathsf{LS}}(a) = 1 - \mathsf{LS}(w_{\delta}, a) = 1 - \frac{-\ln(|(1-0)-0|) - \ln(|(1-1)-0.5|)}{2} \approx 0.65$$
$$\mathsf{U}_{\mathsf{LS}}(b) = 1 - \mathsf{LS}(w_{\delta}, b) = 1 - \frac{-\ln(|(1-0)-0.2|) - \ln(|(1-1)-0.55|)}{2} \approx 0.59$$

Therefore basing Caie's Accuracy Criterion on the logarithmic scoring rule leads to Roy being rationally required to be in credal state a. □

In Theorem 4 we showed that Caie's Accuracy Criterion based on the Brier score led to Roy being rationally required to have credal state b. Therefore we see that Caie's Accuracy Criterion is dependent on the scoring rule chosen.

This is not specific to the logarithmic scoring rule. We can find similar examples for most pairs of scoring rules.[13]

Given this scoring rule dependence, if one accepts Caie's modification to the accuracy criterion one needs to explain what rational constraints accuracy-dominance considerations lead to. There are at least four options.[14] Firstly, one could give arguments for one particular scoring rule and argue that accuracy-dominance considerations require one to minimize inaccuracy with respect to that scoring rule. Secondly, one could take a *subjectivist* approach and argue that for each agent and context there is some particular measure of inaccuracy which is appropriate. Thirdly, one could take a *supervaluationist* approach and argue that the notion of inaccuracy is vague and that any inaccuracy measure satisfying certain conditions is an appropriate precisification of it; to satisfy accuracy dominance considerations one would then have to minimise inaccuracy with respect to at least one appropriate inaccuracy measure. Lastly one could take an *epistemicist* approach and argue that although there is some particular scoring rule which one should be minimising inaccuracy with respect to, we do not know which it is.[15] Each of these ways of dealing with the scoring rule dependence will still lead to the rejection of probabilism since different scoring rules will still lead to rational probabilistic incoherence.

[13] This can be seen by varying the above proof. Choose credal states a and b, where a is closer to $\langle 0, 1\rangle$ using one scoring rule, and b is closer using the other. Then construct a sentence where a and b are the credal states which compete for being the most accurate.

[14] This problem is very closely related to a problem for the traditional accuracy argument which is that there is no credence function that dominates on every measure. This is discussed in [Pettigrew, 2011, section 6.2.2]. Furthermore the ways of dealing with the two problems are similar and these options presented here parallel the options presented in Pettigrew's article.

[15] The disadvantage of this version of accuracy considerations is that an agent does not have the resources to know whether she satisfies the rational requirement or not.

5 Conclusion

In Section 1 we argued that Probabilism isn't to blame for the failure of Weak Rational Introspection, therefore undermining Caie's first argument for rational probabilistic incoherence. In the other sections we showed that Caie's modified accuracy criterion leads to the failure of rational introspection, leads to needless loss on bets and is scoring rule dependent.

In [Caie, 2013] Caie showed that probabilism is incompatible with each of: introspection, a modified accuracy criterion, and Dutch book considerations. What we have shown in this paper is that Caie's modification of the accuracy criterion is incompatible with each of: probabilism[16] (or equally the traditional accuracy criterion), introspection, Dutch book considerations, and an analogous criterion based on a different scoring rule. What we therefore result in is a list of principles which initially seem desirable but where neither probabilism nor the modified accuracy criterion can be accepted without ruling out all the other principles. In fact by giving other examples similar to those in this paper we can show that no two of these principles can be taken together.[17]

This shows that one does not end up with a coherent picture of what is rational for an agent by just rejecting probabilism from one's accepted principles (and keeping the other discussed principles) since there are still incompatibilities remaining. The only way to end up with a coherent picture is to reject all but one of the principles considered. This is an interesting and important feature of Caie's position which was previously unnoticed.

There is a different way to take these results: Without these results one might think that Caie's modification of the accuracy criterion is better than the traditional accuracy criterion because the traditional accuracy criterion conflicts with Dutch book considerations and leads to poor epistemic access. However the results in this paper show that that the modified accuracy criterion Caie proposes also conflicts with Dutch book considerations and leads to poor epistemic access (and is also scoring rule dependent unlike the traditional criterion). This might therefore give one a reason to reconsider the traditional accuracy criterion and suggest that it is in fact the correct way to apply accuracy considerations even when propositions like # are considered. One would still have to say how and why the traditional criterion does appropriately apply in such

[16]This was already shown in Caie's paper.

[17]The introspection and Dutch book criteria that we focused on were written with a rationally permissible operator so Weak Rational Introspection and Weak Loss Minimization are clearly compatible. What we can actually show is that if one strips the "rationally required" and "rationally permissible" operators from the principles and just looks at the desirable characteristic they express (e.g. if the agent is in credal state Cr she has good epistemic access to her own credal state) there is an algebra in which no two of these characteristics are satisfied together. For a single algebra showing the pairwise inconsistencies one can consider:

$$(\zeta) \qquad \begin{aligned} &\mathsf{Cr}_B\rho(\zeta)\leqslant 0.2 \vee (\mathsf{Cr}_B\rho(\zeta)\leqslant 0.35 \wedge \mathsf{Cr}_B\rho(\neg\zeta)\leqslant 0.2)\\ &\vee(\mathsf{Cr}_B\rho(\zeta)\leqslant 0.5 \wedge \mathsf{Cr}_B\rho(\neg\zeta)\leqslant 0.45) \vee (\mathsf{Cr}_B\rho(\zeta)\leqslant \mathsf{Cr}_B\rho(\neg\zeta))\end{aligned}$$

Therefore taking any one of them as rationally required rules out taking any of the others as rationally permissible.

situations so our results haven't shown that this is the correct approach, just that the approach seems more tenable than it seemed before.

References

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