Epistemic Dilemmas, Undermining Scenarios and Determinate Recommendations

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1 Introduction

There are certain scenarios where one's credences undermine their own adoption and lead to seeming epistemic dilemmas, for example:

Ring Toss

> You are playing ring toss at the carnival and are about to throw your ring at the peg. If your credence that you'll get it on the peg is $\geqslant 0.5$, then you'll be tense and miss. If your credence is not $\geqslant 0.5$, then you'll be relaxed and make it. And you know this about yourself. What is epistemically required of you?
> *Closely related to Archer case in Joyce, 2018 or Basketball in Caie, 2013.*

Suppose you have adopted credence 0.2. Then you will successfully get the ring on the peg, and you know this. Having adopted 0.2, your evidence supports adopting credence 1; and 0.2 undermines its own rational adoption. However, were you to have credence 1, then you'd know that you'll fail. So we might conclude that adopting credence 1 recommends adopting credence 0, also undermining its own adoption. Similarly, any credence value you might adopt would undermine its rational adoption.

Following this line of thought, cases like Ring Toss seem to result in epistemic dilemmas where there are no credences you could adopt which would be rational. This is the position argued for in Konek and Levinstein (2019) and Joyce (2018). There are alternatives offered by Pettigrew (2018) and Caie (2013) who say that the way I just spelled out recommendation and underminingness aren't ultimately key for rationality judgements. Their position avoids the epistemic dilemma but results in a picture of rationality which is unattractive and loses intuitive ties between rationality and evidence.[1]

To preserve these desirable features, the intuitive notion of recommendation should be used. But in using this, it seems that we result in epistemic dilemmas. I offer a proposal that still takes this recommendation notion to be central for an account of rationality, but nonetheless allows epistemic dilemmas to be avoided.

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[1]See Carr, 2017. In fact, this rejects almost all intuitive rationality constraints outright (Campbell-Moore 2016, Section 7.2, Campbell-Moore 2015.)

We allow that one's credence may be an indeterminate matter, and it is this indeterminate state that is judged for rationality. When one's credences are indeterminate, it may be indeterminate whether something is recommended. We propose rationality constraints on one's indeterminate credal state which require that one determinately satisfies any determinate recommendations.

Consider being indeterminate between the two extremal values of 0 and 1 in the case of Ring Toss. It is determinately recommended that one's credence be either 0 or 1, but indeterminate which. This epistemic state thus determinately satisfies any determinate recommendations and will be proposed as rational. The epistemic dilemma has thus been avoided.

If one makes the additional assumption that one's credence is determinately adopted, then our constraints exactly agree with the judgements of Konek and Levinstein (2019) and Joyce (2018). However, we drop this requirement that one's epistemic state be a determinate matter and we move the focus from recommendations to determinate recommendations. This allows us to avoid the epistemic dilemmas.

The formal background for this proposal is from relating such cases to the liar paradox and implementing work accounting for the liar paradox, which is a sentence:

`Liar`: `Liar` is not true.

In particular, this account is motivated by a supervaluational variant of the prominent account developed in Kripke (1975), using some of the interpretation from McGee (1989).[2] In this, one considers truth to be vague, or indeterminate. Due to the liar sentence, we cannot have φ is true iff $\mathrm{Tr}\ulcorner\varphi\urcorner$ is true (without giving up classical logic). We therefore also cannot have that it is determinate that φ is true iff $\mathrm{Tr}\ulcorner\varphi\urcorner$ is true. However, we can obtain "narrow scope" variants such as: if φ is determinately true, then so is $\mathrm{Tr}\ulcorner\varphi\urcorner$. McGee (1989, 1990) (following Kripke, 1975) argues that it is such principles which are important for an adequate account of truth.[3]

Similarly here, we cannot maintain that it is determinate that one's credences follow the recommendations. That is the challenge offered by cases such as Ring Toss. But we can obtain closely related constraints by having that one's credences determinately follow any determinate recommendations. So, for example, one's credences must determinately follow any determinate evidence. We propose that these narrow scope constraints are the ones that are important for judging one's rationality; and thus that the epistemic dilemmas such as that of Ring Toss can be avoided.

In this paper, we will present these narrow scope constraints and investigate the resultant picture of rationality. We propose our account to be an attractive general account of rationality in indeterminate settings, which also allows epistemic dilemmas in cases like Ring Toss to be avoided.

[2] Campbell-Moore (2021) can be consulted for further comments on the connections and formal account, though there are a few differences between the setup of that paper and what is proposed here.

[3] McGee in fact talks about *definite* truths.

The paper is structured as follows:

In section 2 we discuss the recommendation notion applying to credal states, where what is recommended depends on the credal state adopted.

In section 3 we discuss indeterminate credences and the epistemic requirements on them. Our account is supervaluational in spirit: what is required of one's indeterminate credence depends on the recommendations that govern the members of the credal set, or precisifications of the indeterminate credences. We propose two rationality constraints on one's indeterminate credal state:

Recommendation Constraint
Every determinate recommendation should be determinately satisfied.

Acceptability Constraint
Every determinately satisfied property should be determinately acceptable.

In section 4 we discuss the consequences of applying these constraints in various cases. We are able to obtain narrow scope variants of the various desirable principles. For example, that your credences should determinately follow any determinate evidence.

We then discuss the two main remaining challenges with cases where epistemic dilemmas might rearise.

Firstly, in section 5, I observe that to avoid epistemic dilemmas, we might need to move to a slightly more complicated picture of indeterminate belief, where we focus directly on the definite features of one's credence and there may be no credences whatsoever that have all the definite features.

Secondly, in section 6, we discuss a revenge-style challenge: Maybe what happens depends on my *indeterminate* credences in an undermining way. In fact, Recommendation and Acceptability Constraint can still be satisfied in such a case. The case highlights, though, that the move to indeterminacy wasn't the only important move, it was also important that we move from looking at recommendations to determinate recommendations.

2 Recommendations

We start with a bit of setup: A credence function is a (usually probabilistic) function assigning a numerical degree of belief between 0 and 1 to each relevant proposition. In this paper we are often only interested in a single proposition and are then just talking about your credence in this particular proposition, which is a value between 0 and 1. But our official account works more generally with credence functions more generally.

2.1 The recommendation notion

In the Ring Toss case, we are just interested in your credence that you'll get the ring on the peg. If you have credence 0.2, then this creates evidence that tells you that it will happen for certain. The only credence that is compatible with this newly created evidence is credence 1. If you have credence 0.2, credence 1 is uniquely epistemically acceptable. More generally, for the Ring Toss case:

- If you have credence $x \leqslant 0.5$, then credence 1 is uniquely (epistemically) acceptable.

- If you have credence $x > 0.5$, then credence 0 is uniquely (epistemically) acceptable.

We call this sort of thing a *recommendation notion*. It specifies which credences are epistemically acceptable depending on which credences are adopted. Formally it is just a function from credence functions to (non-empty) sets of credence functions. Often we are considering cases where there is a uniquely acceptable credence, so the recommendation notion is just a function from credences to credences. But we want our account to also apply when multiple credences are acceptable.

Our account will importantly rely on how one spells out any particular case with such a recommendation notion.

A gloss on this recommendation notion is that it encodes which credences are supported by the evidence; where what evidence you have may depend on the adopted credence. But some readers may wish to spell it out in different ways. For example, it could be grounded in considerations such as maximising the accuracy of one's credences, where we would say that a particular credence is acceptable iff it is not accuracy-dominated, or perhaps iff it maximises estimated accuracy. Our account naturally goes with an implementation of this following Konek and Levinstein (2019), Joyce (2018) rather than the "consequentialism" of Pettigrew (2018), Caie (2013). This can also be applied in cases where what is acceptable doesn't depend on the adopted credences. In that case, I nonetheless propose that rationality applies to ones (possibly) indeterminate states in accordance with Recommendation and Acceptability Constraint, though it can be rational to determinately have some particular credence, unlike cases like Ring Toss, one has to be indeterminate to be rational.

The kinds of cases that have been discussed in this area are typically given in a way where the intended recommendation notion is clear. For example the case of Ring Toss is described exactly so as to give this recommendation notion.[4]

We assume that we always have a recommendation notion given. It should encompass anything one thinks is intuitive about the case, which will then allow us to get versions of these intuitive results for rationality.

There is a key assumption about the recommendation notion that we make: we assume that whatever credence has been adopted, some credence will be acceptable (although it may not be the initially adopted credence). We are thus ruling out, or not considering, epistemic dilemmas that may arise for different reasons. The only kinds of potential epistemic dilemmas we consider are ones where what is acceptable depends on what is adopted in an undermining way.

2.2 Recommended properties

To state our rationality requirements we will use facts about which properties are acceptable or recommended. Properties are understood extensionally as characterised by a set of credence functions. Suppose that you have credence 0.2. Then credence 1 is uniquely acceptable; and so it is recommended to have credence 1, i.e., the (credal) property of being identical to 1 is a recommended property. It is also recommended to have a credence $\geqslant 0.5$; after all, the uniquely acceptable credence of 1 is $\geqslant 0.5$. In general:

[4] The recommendation notion may additionally require one to have settled on a causal or evidential implementation of such considerations. (Greaves, 2013).

- A property of one's credences is *recommended* iff every acceptable credence has that property.

Note that this notion of recommendation depends on which credence has been adopted, because that will affect which credences are acceptable.

If there were multiple acceptable credences, c_1 and c_2, then it is recommended to have a credence in the set $\{c_1, c_2\}$, that is, to have either credence c_1 or c_2. It is not, however, recommended to have any particular one of these, say credence c_1, itself. Having credence c_1 would go beyond the recommendations; it is acceptable to have credence c_1, but not recommended. It is also acceptable to have a credence in $\{c_1, c_5\}$, by virtue of c_1 being acceptable. We say:

- A property of one's credences is *acceptable* iff there is some credence with that property that is acceptable.

Note that a property being acceptable does not imply that every credence with that property is acceptable, just that at least one is.

'Recommended' should be understood by analogy to 'required', 'acceptable' should be understood by analogy to 'permitted'. We use these alternative terms because propose that rationality applies to one's indeterminate credal state, whereas this recommendation notion governs the precisifications of one's indeterminate credal state, the members of your credal set. However, in our general account, they will line up when your epistemic state is determinately adopted.

3 Indeterminacy

As we noted in section 1, the recommendation notion of Ring Toss is such that no credence can be adopted and satisfy the resultant recommendations. No credence is acceptable according to itself. So we are in epistemic dilemma territory. However, we consider moving to a framework where one's credences may be indeterminate.

3.1 Indeterminate credences

It may be indeterminate what your credence is. There are some determinate matters, for example, perhaps it is determinate that your credence in a particular proposition is $\geqslant 0.5$, but it may be indeterminate whether it is identical to 0.8 or not.

We can formally think of your indeterminate credences with a set of credence functions (usually probabilistic), each of which assigns a numerical degree of belief between 0 and 1 to each relevant proposition. When we are only interested in a single proposition, we can then think just of a set of numerical values, e.g., $\{0, 1\}$.

A determinate credal state is a special case: the singleton $\{b\}$.

The members of one's credal set are the functions that are compatible with all the determinate properties. We will also sometimes talk about these as 'precisifications' of your indeterminate credal state.

Indeterminate probabilities is a model of belief that is independently popular (see, e.g., Joyce, 2010, Bradley, 2015), though some authors working with indeterminate (or "imprecise") probability would not accept the heavy reliance on the notion of indeterminacy and the supervaluational approach that this paper takes.

In section 5, we will further discuss the picture of indeterminate belief, and I will propose allowing for indeterminate beliefs that are not given by a credal set. These are cases where the determinate judgements are infinitely incompatible. However, for the main part of this paper, we consider indeterminate beliefs using credal sets to keep discussions clearer and because it is one of the most prominent models of indeterminate probabilities.

3.2 The rationality constraints on one's indeterminate credences

We propose that it is one's indeterminate epistemic state which is rational or irrational. In our account, the rationality constraints on the indeterminate believer depend on the notion of recommendation, which governs the precisifications of one's indeterminate credal state.

A key idea is that when one's credences are indeterminate, it is also indeterminate what is recommended. For example, if b_1 recommends having c_1 and b_2 recommends having c_2, then if your credences are indeterminate between b_1 and b_2, i.e., you have $\mathbb{B} = \{b_1, b_2\}$, then it is determinate that c_1 or c_2 are recommended, but indeterminate which. (See fig. 1.)

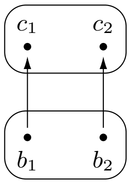

Figure 1: The arrows describe what is evaluated as acceptable each credence: b_1 evaluates c_1 as uniquely acceptable and b_2 evaluates c_2 as uniquely acceptable. Our two constraints mean that $\{b_1, b_2\}$ evaluates $\{c_1, c_2\}$ as uniquely permissible.

We impose the rationality constraint that your indeterminate credences should determinately satisfy any determinate recommendations.

Recommendation Constraint

Every determinate recommendation should be determinately satisfied.

So in this case, since the property of being either c_1 or c_2, that is the property whose extension is $\{c_1, c_2\}$, is determinately recommended, it should be determinate that your credence be in that set; that is you should have some indeterminate credences $\mathbb{C} \subseteq \{c_1, c_2\}$.

As we have just applied this, we have considered holding fixed some adopted indeterminate credences, $\mathbb{B}$, and evaluating some, possibly different, credences, $\mathbb{C}$. We use $\mathbb{B}$ for considering what is determinately recommended and $\mathbb{C}$ for considering what is determinately satisfied. This is a more general application of Recommendation Constraint. When considering if epistemic dilemmas remain, we want to find some indeterminate credences which can be rationally adopted. Talking in terms of evaluation, this means that they should evaluate themselves as permissible.

We also offer a further constraint, Acceptability Constraint, offering something of a converse to Recommendation Constraint.

Acceptability Constraint
Every determinately satisfied property should be determinately acceptable.

In our example case, Acceptability Constraint means you should have $\mathbb{C} \supseteq \{c_1, c_2\}$. The argument to see this is a bit more involved. Consider the property of being not identical to c_1. If it's determinate that your credence is not identical to c_1, i.e., if $c_1 \notin \mathbb{C}$, then this property would have to be determinately acceptable. But when we hold fixed that $\mathbb{B} = \{b_1, b_2\}$ is adopted, this is not determinately acceptable, after all, b_1 thinks that the only acceptable credence is c_1, so b_1 thinks that being not identical to c_1 is unacceptable; and thus the property of being not identical to c_1 is not determinately acceptable. So any $\mathbb{C}$ with $c_1 \notin \mathbb{C}$ fails Acceptability Constraint as evaluated by $\{b_1, b_2\}$. Similarly for c_2.

So the combination of these two constraints mean that if you have the indeterminate credal state $\{b_1, b_2\}$, then you should have $\{c_1, c_2\}$. That is, $\{b_1, b_2\}$ evaluates $\{c_1, c_2\}$ as uniquely permissible. Recommendation Constraint gets us that you should have $\mathbb{C} \subseteq \{c_1, c_2\}$ and Acceptability Constraint gets us that you should have $\mathbb{C} \supseteq \{c_1, c_2\}$.

Recommendation Constraint means that the indeterminacy shouldn't be too wide, Acceptability Constraint means that the indeterminacy shouldn't be too narrow.

3.2.1 Applying to Ring Toss

How do Recommendation and Acceptability Constraint apply in the case of Ring Toss? We will see that it is rational to be indeterminate between 0 and 1. That is, you can rationally adopt $\mathbb{B} = \{0, 1\}$ in accordance with our constraints. It evaluates itself as permissible.

Suppose you have adopted $\mathbb{B} = \{0, 1\}$. It is determinately recommended that your credence be either 0 or 1. This is because both 0 and 1 recommend that one's credence be either 0 or 1, though they disagree on which. This determinately recommended property is determinately satisfied by $\{0, 1\}$.

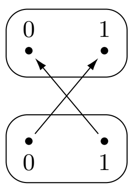

Figure 2: For Ring Toss, $\{0, 1\}$ evaluates itself as acceptable

Stronger properties, such as being identical to 0 are not determinately recommended, as one of the members, 0, does not recommend it. So no stronger property should be determinately satisfied and $\{0, 1\}$ is permissible in accordance with Recommendation Constraint.

Acceptability Constraint is also satisfied: the determinate property of being 0 or 1 is determinately acceptable. 0 evaluates it as acceptable by virtue of 1 being acceptable; 1 evaluates it as acceptable by virtue of 0 being acceptable.

So $\{0, 1\}$ can rationally be adopted in the Ring Toss case, and the seeming epistemic dilemma is avoided.

In fact, for the recommendation notion of Ring Toss, $\{0, 1\}$ is the only state that can be rationally adopted.[5]

3.2.2 Another abstract example

Our constraints are also applicable when recommendation leaves options open, that is, when there can be multiple acceptable credences. We consider such an example here.

Suppose b_1 evaluates both c_1 and c_2 as acceptable, and b_2 evaluates c_3 as uniquely acceptable. If we hold fixed that one's indeterminate credal state is given by $\mathbb{B} = \{b_1, b_2\}$, the indeterminate credences that satisfy these constraints are $\{c_1, c_2\}$, $\{c_1, c_3\}$ and $\{c_1, c_2, c_3\}$.[6] (See fig. 3.)

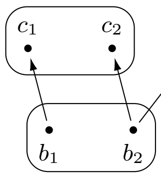

Figure 3: $\{c_1, c_2\}$ is evaluated as acceptable according to $\{b_1, b_2\}$; so is $\{c_1, c_3\}$ and $\{c_1, c_2, c_3\}$.

Recommendation Constraint requires that one's credence is determinately in $\{c_1, c_2, c_3\}$, i.e., the indeterminate state is a subset of this. To satisfy Acceptability Constraint, c_1 needs to be in the set, and so does at least one of c_2 and c_3. The combination of the two constraints, then, give us $\{c_1, c_2\}$, that $\{c_1, c_3\}$ and $\{c_1, c_2, c_3\}$ as rationally acceptable.

3.2.3 Summarising, and alternative forms

To summarise, the rationality constraints on one's indeterminate attitudes are given by:

Recommendation Constraint

Every determinate$_{\mathbb{B}}$ recommendation must be determinately$_{\mathbb{C}}$ satisfied.

Acceptability Constraint

Every determinately$_{\mathbb{C}}$ satisfied property must be determinately$_{\mathbb{B}}$ acceptable.

We have also indicated in these principles how they should be used when a fixed adopted indeterminate state, $\mathbb{B}$, is used to evaluate another indeterminate state, $\mathbb{C}$.

[5] Whatever indeterminate credences are held, it is determinately recommended to be either 0 or 1. So it has to determinately be 0 or 1 to satisfy Recommendation Constraint. The only other epistemic states where it is determinately either 0 or 1 are $\{0\}$ and $\{1\}$. But if $\{1\}$ is adopted, then $\{0\}$ is required; and if $\{0\}$ is adopted, then $\{1\}$ is required. So Recommendation Constraint means that only $\{0, 1\}$ can rationally be adopted.

[6] If we suppose that actually $c_1 = b_1$ and $c_2 = b_2$, then this shows us that Acceptability Constraint adds an additional restriction even for self-acceptance.

What these entail in particular cases depends on the given recommendation notion. In section 4 we will discuss some principles we obtain as a consequence of it.

Recommendation Constraint is the most important of these two constraints. Acceptability Constraint is less well supported, but it is natural and has some desirable consequences, so we adopt it.

There are some other ways we could have given these principles.[7] Recommendation Constraint is equivalent to:

> Every member of $\mathbb{C}$ is acceptable according to some member of $\mathbb{B}$.

and Acceptability Constraint is equivalent to:

> Every member of $\mathbb{B}$ evaluates some member of $\mathbb{C}$ as acceptable.

There is also a further equivalence for Acceptability Constraint[8]

> If some member of $\mathbb{B}$ makes some recommendation, then some member of $\mathbb{C}$ satisfies it.

To evaluate whether a state can be rationally adopted, we apply these with $\mathbb{B}$ and $\mathbb{C}$ identical.

When we assume that one has a determinate credence, our constraints simply say that to be rational, this determinately adopted credence must be self-acceptable. But it can also allow some indeterminate credences to be rational.

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[7]

Proof of the equivalence for Recommendation Constraint . P is recommended by b iff $P \supseteq \{y \mid y$ is acceptable according to $b\}$. P is a determinate recommendation iff

$$P \supseteq \{y \mid \ y \text{ is acceptable according to every } b \in \mathbb{B}\}$$

So, Recommendation Constraint just requires that

$$\{y \mid \ y \text{ is acceptable according to every } b \in \mathbb{B}\}$$

is determinately satisfied, that is, every $c \in \mathbb{C}$ is in this set; which is what our reformulation states. □

Proof of the equivalence for Acceptability Constraint . P is acceptable according to b iff $P \cap \{y \mid y$ is acceptable according to $b\} \neq \varnothing$.

Acceptability Constraint requires:

$$P \supseteq \mathbb{C} \implies P \cap \{y \mid y \text{ is acceptable according to } b\} \neq \varnothing \text{ for every } b \in \mathbb{B}.$$

which can be reduced to:

$$\mathbb{C} \cap \{y \mid y \text{ is acceptable according to } b\} \neq \varnothing \text{ for every } b \in \mathbb{B}.$$

which is what our reformulation states. □

[8]To see this is equivalent, with abbreviations, one might reason:

Acceptability Constraint:	Det P	$\Longrightarrow$	Det Acc P
iff	$\neg$ Det Acc P	$\Longrightarrow$	$\neg$ Det P
iff	$\neg$ Det $\neg$ Rec $\neg P$	$\Longrightarrow$	$\neg$ Det P
iff	SomePrec Rec $\neg P$	$\Longrightarrow$	SomePrec $\neg P$

One might use the term "indeterminate" as a shorthand for "some member" but this might mislead one to thinking that it excludes being determinate.

3.3 A stronger constraint that we do not adopt

There is a stronger version of Recommendation Constraint which we will not require for rationality:

> It must be determinate that: every recommendation is satisfied.

The difference is the scope of 'determinate'. In our Recommendation Constraint, 'determinate' has narrow scope; the determinate recommendations must be determinately satisfied. In this stronger constraint, 'determinate' has wide scope, so the members that are doing the recommendations have to be coordinated with the members that are satisfying them.

This strengthening then means that in cases like Ring Toss, it cannot be satisfied. No credences whatsoever satisfy their own recommendations, so it certainly can't be determinate that one's credences satisfy their recommendations.

Whilst we propose that the stronger version is not required for rationality, an interlocutor might want to hold on to it. Such an interlocutor would then be committed to epistemic dilemmas in cases like Ring Toss. They might nonetheless be interested in investigating the consequences of the weaker principles of Recommendation and Acceptability Constraint. Whilst this interlocutor would not accept such epistemic states as rational, they have some "thumbs up" which the irrational ones which do not satisfy these principles do not have. This may nonetheless be interesting to our interlocutor who holds on to the wide-scope variants and thus cannot avoid the epistemic dilemma.

For this paper, we take it that the stronger constraints are not requirements of rationality, and that Recommendation and Acceptability Constraint suffice, and thus, that the epistemic dilemmas are avoided.

4 Applying this Account

We have proposed that judgements of rationality apply to one's indeterminate epistemic state, and that the rationality constraints are given by Recommendation and Acceptability Constraint.

This doesn't say all there is to be said about rationality because there is a lot being encoded in the recommendation notion which governs the credal members, the precisifications of the indeterminate credal state. My suggestion is that most discussions about rationality should be understood instead as discussions about what is recommended (must it follow the evidence, etc). But once that has been settled, Recommendation and Acceptability Constraint are the central constraints to then determine whether one's indeterminate epistemic state is rational or not.

We now consider what happens when we apply these constraints and considerations in various cases, and investigate the resultant principles that we obtain.

4.1 Extremal Bad Navigator and Evidence

Consider the following case:

Extremal Bad Navigator

You've come to a crossroads and are wondering whether you need to turn left or right to get to your hotel. You know you're an extremely bad navigator. In particular, if you adopt credence $\geqslant 0.5$ that it's to the left, then you know for sure that it's to the right, and if you adopt credence < 0.5 that it's to the left, then you know for sure that it's to the left. (Related to an example in Egan and Elga, 2005)

Here, the credence you adopt results in additional evidence regarding the location of your hotel, and in fact does so in an extremal and undermining way. If you have credence $\geqslant 0.5$, then you know it's to the right, so credence 0 is epistemically recommended. If not, then you know it's to the left and credence 1 is recommended. This is exactly the same recommendation function that was obtained in the case of Ring Toss.

Since our proposed rationality constraints of Recommendation and Acceptability Constraint only depend on this recommendation notion, they will result in the same constraints in the cases of Extremal Bad Navigator and Ring Toss. That is, to be rational, one's credence should be indeterminate between the two extremal values of 0 or 1.

The cases differ in the phenomenon underlying the recommendation notion: in the case of Ring Toss, your credence had causal power, whereas for Bad Navigator, it simply generates additional evidence, however the recommendation notion we obtain is the same and thus Recommendation and Acceptability Constraint apply analogously, obtaining that the only epistemic state that can be rationally adopted is the indeterminate state $\{0, 1\}$.

Extremal Bad Navigator is a modification of a case provided by Egan and Elga (2005) where one's all-things-considered judgements anti-correlate with the truth. Egan and Elga say in such case, one should suspend judgement. They say:

> "Moral: When one becomes convinced that one's all-things-considered judgments in a domain are produced by an anti-reliable process, one should suspend judgment in that domain." Egan and Elga (2005, p83)

There are some relationships between having indeterminate credences and suspending judgement. So whilst our proposal is different to that of Egan and Elga, it does bear some similarity in spirit.

We can also use this example to consider what epistemic rationality principles are obtained. When we were considering rationality applying to one's credence, ignoring any potential indeterminacy, we might have expected that we have the rationality principle:

EvPrec

If E is part of your evidence, then your credence in E should be 1. ✗

But the Extremal Bad Navigator case causes difficulties for this principle. We can maintain the idea of this principle as a putative epistemic requirement and include it in an analysis of what is *recommended*. If E is part of your evidence, then it is recommended that you have credence 1 in E. Then when we move to considering rationality which accommodates potential imprecision, we use Recommendation Constraint and obtain:

EvDet

If E is determinately part of your evidence, then you should determinately have credence 1 in E. ✓

This constraint is satisfiable. I propose it contains the key important content of the original unsatisfiable principle, EvPrec.

4.2 Extremal Promotion and Chance

A range of cases where one's credences affect the evidence one has were given by Greaves (2013). Her examples are formulated involving "chance". Consider for example the following case, which is a variant on her Promotion case (Greaves, 2013, p. 915)

Extremal Promotion

> "Alice is up for promotion. Her boss, however, is a deeply insecure type: he is more likely to promote Alice if she comes across as lacking in confidence. Furthermore, Alice is useless at play-acting, so she will come across that way iff she really does have a low degree of belief that she's going to get the promotion." Specifically, if she has credence $\geqslant 0.5$, then the chance she'll be promoted is 0, and if not, then the chance she'll be promoted is 1. *This example is considered by Konek and Levinstein (2019).*

The recommendation notion in this case again matches that of Ring Toss, now being because of credences affecting the chances.

No credence you could assign matches the chances. Were you to assign 0, or 0.2, or anything that is not $\geqslant 0.5$, then the chance you'll get the promotion is 1. So these values don't match the chances. And similarly if you assign credence 0.8, or 1, or anything that is $\geqslant 0.5$ then the chance you'll get the promotion is 0. So there is no credence value whatsoever which you could adopt and it would match the chances.

However, Recommendation and Acceptability Constraint can apply and we can obtain a match between one's indeterminate credal state and the indeterminate chances. If you adopt the extremal indeterminate credal state $\{0, 1\}$, then it is also indeterminate whether the chance is 1 or 0. So we obtain a match between the indeterminate credences and the indeterminate chances. The matching is not a member-by-member, or precisification-by-precisification, match. Each credence is maximally far away from the chances it brings about, but the indeterminate credences match the indeterminate chances.

For Extremal Promotion, the indeterminate state $\{0, 1\}$ can be rationally adopted and the epistemic dilemma is avoided.

What principles do we then obtain regarding deference-to-chance? We may have usually expected:

ChPrec1

> If you are certain that the chance is x, then you have credence x. ✗

This should now be taken as a principle regarding what is *recommended* and we then obtain:

ChDet1

> If you are determinately certain that the chance is x, then you should determinately have credence x. ✓

We could also apply this not by thinking about individual propositions and chance values but about the entire chance function. If you are determinately certain that the chance function is some particular function, ch, then your credence should determinately be ch.

This doesn't actually tell us much about Extremal Promotion when you are indeterminate between 0 and 1 because the chance is also indeterminate. We can describe a more general principle that would also cover this case:

ChDet1′

> If you are determinately certain what the chances are, then your indeterminate credences should match the indeterminate chances. ✓

Just being determinately certain that the chance has some particular property does not mean that your credences must also determinately have that property, unless you determinately know what the chances are. For example, it is not a putative epistemic constraint that if you are certain that the chance is either 0 or 1 then you should have credence 0 or 1. A middling response might be perfectly rational. However, we would usually expect this to hold for *convex* properties. For example, that the chance is in an interval $[a, b]$. Or that the chance of one proposition is higher than another. That is, one would usually expect:

ChPrec2

> If P is a convex property, and you are certain that the chance function has property P, then you should have credence satisfying P. ✗

By imposing this as a constraint on recommendation, we obtain from Recommendation and Acceptability Constraint:

ChDet2

> If P is a convex property, and you are *determinately* certain that the chance function has property P, then you should *determinately* have credence satisfying P. ✓

For example, if you determinately know that the chance of something happening is $\geqslant 0.5$, then you should determinately have credence $\geqslant 0.5$.

4.3 Credal Liar

There is a further case that's been mentioned in the literature which leads to the same notion of recommendation as Ring Toss.

Consider the sentence, `CredLiar`:

> `CredLiar`: Your credence in `CredLiar` is not $\geqslant 0.5$.

In this case, by adopting a particular credence, you semantically affect the truth value of `CredLiar`, rather than in the case of Ring Toss where it is a causal process. But the same reasoning exactly follows. Any credence value you could assign would undermine itself. But the credal state which is indeterminate between 0 and 1 satisfies our demands and can be admitted as a rational response.

There is then a nice match between your credence and its truth value. It is indeterminate whether `CredLiar` is true or false. And you have a credal state that is indeterminate between 1 and 0.

Do we obtain this as a general constraint? If you know that some particular proposition is indeterminate, should your credence be indeterminate between 0 and 1? With a very careful formulation, this will come out as a consequence of Recommendation Constraint:

If, in every precisification you are certain of its truth value, and in some precisifications is true and some false, then your credence must be indeterminate between equalling 0 and equalling 1.

This does, though, require the assumption that you are certain of its truth value in each precisification for reasons similar to the care we took in the case of chances.

4.4 Leap

Each of the cases we have considered so far have the same recommendation notion, and they all result in the way to satisfy Recommendation and Acceptability Constraint being to have the extremal indeterminate state of being indeterminate between credence 0 and 1.

We now move to considering cases which have different recommendation notions and will then result in different rational responses.

Leap

"Bob stands on the brink of a chasm, summoning up the courage to try and leap across it. Confidence helps him in such situations: specifically, for any value of x between 0 and 1, if Bob attempted to leap across the chasm while having degree of belief x that he would succeed, his chance of success would then be x."
Greaves (2013)[9]

In this case, every particular credence you could assign will evaluate just itself as acceptable. And thus, as described in section 3.2, Recommendation and Acceptability Constraint mean that any indeterminate credal state evaluates itself as uniquely permissible. In the leap case, then, any indeterminate credal state, $\mathbb{B}$, can be rationally adopted, including when it is determinate what your credences are: $\{b\}$.[10]

4.5 Normal cases

In all the cases we have considered so far, adopting some credence generates additional information. Most epistemic scenarios aren't like this. Consider a more typical case:

Rain

The credence that you adopt that it is going to rain tomorrow provides no additional evidence about the likelihood of rain.

How do our considerations apply to such cases?

There are two ways one might implement the notion of recommendation in such a case (see also fig. 4).

[9] See also James (1897, p.59).

[10] For the purposes of this paper, we are considering Recommendation and Acceptability Constraint to suffice for rationality. In discussing this case, Joyce (2018) adds further constraint to rule out some such states as irrational. It is not clear exactly how such constraints would apply in our indeterminate setting, but we are open to there being further constraints on rationality so long as they do not reduce the situation to one of an epistemic dilemma.

Figure 4: Two possible implementations of recommendation in a case like Rain.

The first of these says that in these normal cases, the credence you have adopted doesn't affect what is recommended. It is only in cases where one's adopted credence changes the evidence that the recommendations are altered. So in a normal case like this, a credence of 0.2 is either acceptable or not, and this doesn't depend on what credence has been adopted. Following this implementation, then, an imprecise credal state of $\mathbb{C}$ satisfies our Recommendation and Acceptability Constraint (according to any $\mathbb{B}$) iff its members are themselves acceptable in this non-relative sense.

The second natural implementation follows the idea that recommendations should follow the epistemic evaluations of the adopted credences. Suppose you've adopted the credence value 0.8 that it will rain tomorrow. What is now recommended of you? A credence of 0.8 evaluates itself as epistemically better than any other particular credence.[11] If recommendations follow such evaluations, then 0.8 is uniquely acceptable according to 0.8. This implementation says that any credence will recommend itself; at least if it is in the range of appropriate credences, such as being probabilistic or appropriately deferring to chances. The implementation will typically match the first implementation by having that the range of credences that were acceptable in the non-relative sense under the first implementation will here be the one's that evaluate themselves as uniquely acceptable; with the others always evaluating themselves as unacceptable. What we then result in is a situation very close to that of Leap and any indeterminate credence evaluates itself as uniquely acceptable. This is a nice consequence.[12]

So, whichever way one implements this, any indeterminate state $\mathbb{B}$ can be rationally adopted in accordance with Recommendation and Acceptability Constraint; so long as the individual members of $\mathbb{B}$ are in a range of appropriate credences and thus will evaluate themselves as appropriate. (In the former implementation they will also evaluate others as appropriate, in the latter, they will uniquely evaluate themselves as appropriate.)

4.6 Promotion

We obtain a different recommendation notion when we consider the original Promotion case by Greaves:

Original Promotion

> "Alice is up for promotion. Her boss, however, is a deeply insecure type: he is more likely to promote Alice if she comes across as lacking in confidence. Furthermore, Alice is useless at play-acting, so she will come across that way iff she really does have a low degree of belief that she's going to get the

[11] At least if 0.8 is in accordance with any other information one has in the situation. The difference between the normal cases and the other cases we've been considering is that in this case the information is not dependent on the credences that have been adopted.

[12] Compare this to Builes et al. (2020) which permits contraction but not expansion. Their formalism can be related to mine by the notion of a guess being licenced as it not being determinately unacceptable.

promotion. Specifically, the chance of her getting the promotion will $1-x$, where x is whatever degree of belief you choose to have in the proposition that she will be promoted."
Greaves (2013)

The recommendation notion this leads to says that if you have adopted credence x, credence $1-x$ is uniquely acceptable.

In this case, determinately adopting 0.5 is rational according to our constraints. But there are also indetermiante credal states which could be rationally adopted and satisfy Recommendation and Acceptability Constraint, for example, $\{0.2, 0.8\}$ or $[0.2, 0.8]$. Any set which is symmetric, which contains $1-x$ iff it contains x, is a indeterminate credal state which can be rationally adopted according to Recommendation and Acceptability Constraint.

The key important difference between Extremal Promotion and Original Promotion is that Original Promotion gives us a continuous notion of support. In Original Promotion, the chances continuously depend on one's credences, whereas in Extremal Promotion there is a point of discontinuity at 0.5.

It is a general fact that whenever the chances depend continuously on one's adopted credences, then there will be some credence which would equal the resultant chances were it to be adopted.[13] And thus, there would be some rational determinate credal state. This doesn't mean that one's credences *must* be determinate. There will also typically be indeterminate credences which satisfy Recommendation and Acceptability Constraint.

When the chances depend on the credences in a discontinuous way, such as in Extremal Promotion, there might be no determinate credences that can be rationally adopted. Usually, though, there will be indeterminate credences that can (see section 5 for the precise status of this claim).

4.7 Imps and Bribes

Greaves also provides us with the following case, where you are offered an *epistemic bribe*:

Imps

You are in the garden and the sun is shining brightly on you. There are ten impish children in a playhouse. Whether each child comes out to play depends in part on your credence that that it is sunny. In particular the chance that each child comes out to play is $(1-\frac{x}{2})$, where x is your credence that it is sunny today.

This case offers an epistemic bribe: if you deny the manifest, and adopt credence 0 that it is sunny, you can ensure perfect credences in each proposition regarding the children playing. A consequentialist implementation of epistemic value considerations, as given by Caie (2013), Pettigrew (2018), would say that

[13]The Intermediate Value Theorem gives us that any continuous function $f : [0,1] \to [0,1]$ has a fixed point. For single-proposition cases this is enough for our claim. More generally, our claim follows from the Brouwer fixed point theorem which generalises this to continuous functions on convex, compact spaces, such as the set of probability functions.

Joyce claims that $x = (1-x)^{1/2}$ has no solutions in $[0,1]$, and thus calls this the "chance paradox". But it has the solution of $x \approx 0.618$. A solution to $x = (1-x)^{1/2}$ requires that $x^2 + x - 1 = 0$ instead of, as Joyce claims, $x^2 - x + 1 = 0$.

one is required to take such epistemic bribes.[14] However, Joyce (2018), and Konek and Levinstein (2019) argue that it is never rationally permissible to take such bribes.

Our account also says you should refuse to take the epistemic bribe. Whatever credence you have adopted, it is recommended to have credence 1 that it is sunny. And so, to satisfy Recommendation Constraint you have to determinately have credence 1 that it is sunny. You must also, then have credence $1/2$ that each child will come out to play.

More generally, as we discussed in section 4.1, our account maintains:

> If E is determinately part of your evidence, then you should determinately have credence 1 in E. ✓

So epistemic bribes that ask us to deny determinate evidence must be rejected.

4.8 Basketball

In our final case, we consider another case that is closely related to Ring Toss.

Basketball

> You are an basketball player in the process of taking your free throw. You will make the shot iff you are less confident that she will make it than that she will miss.
> *(Caie, 2013).*

Here it is most natural to consider both your credence that you'll make it and that you'll fail.

Suppose you have adopted some credence x that you'll make it, and y that you'll fail, which we can write as $\langle x, y\rangle$. If $x < y$, then you'll make it, and you know that, so this would evaluate $\langle 1, 0\rangle$ as uniquely acceptable. If $x \geqslant y$, then you'll miss, and you know that so $\langle 0, 1\rangle$ would be uniquely acceptable. This is very similar to the Ring Toss case, and the only indeterminate credences which then satisfy our constraints are being indeterminate between $\langle 1, 0\rangle$ and $\langle 0, 1\rangle$.

5 A Challenge: The Spring Case

We have proposed Recommendation and Acceptability Constraint as the key constraints on rationality and investigated their consequences. This has allowed epistemic dilemmas to be avoided in cases such as Ring Toss. Can all epistemic dilemmas be avoided? Are there any recommendation functions where Recommendation and Acceptability Constraint cannot be satisfied by any indeterminate attitudes? Unfortunately, at least when we assume that those indeterminate attitudes are given by a credal set, then answer is yes, sometimes Recommendation and Acceptability Constraint cannot be satisfied. However, given a more general model of indeterminate belief, one can show that Recommendation and Acceptability Constraint are always satisfiable. That is, for any recommendation notion there is some indeterminate epistemic state that is rational in accordance

[14] Though Joyce and Weatherson (2019) argue that in fact the consequentialist shouldn't take the bribe in this particular case because when we consider one's credence in a whole Boolean algebra, taking the bribe reduces one's overall epistemic value.

with Recommendation and Acceptability Constraint. So such epistemic dilemmas can be avoided in generality.

Consider, for example:

Spring

> You know that you're always twice as confident as you should be in this type of situation. Except you also know that a credence value of 0 would be wrong, then you are certain that it will happen.
>
> I.e, if you have non-zero credence, x, then it is recommended to have credence $x/2$; and if you have credence 0, then 1 is recommended.

In this case, every credence is undermining. For example:

$$\begin{array}{ccc} 1 & \text{recommends} & 1/2 \\ 1/2 & \text{recommends} & 1/4 \\ 1/4 & \text{recommends} & 1/8 \\ & \text{etc} & \\ 0 & \text{recommends} & 1 \end{array}$$

There is also no indeterminate credal state given by a credal set which is compatible with Recommendation Constraint. To illustrate this, observe that adopting $(0,1]$ would lead to the requirement to have $(0,1/2]$, as each non-zero x recommends $x/2$. But adopting $(0,1/2]$ would lead to the requirement to have $(0,1/4]$. And this would lead to $(0,1/8]$, and so on. Each time you try to satisfy the determinate recommendation, a more restrictive determinate recommendation is obtained. A more general argument is relegated to a footnote.[15]

There is a key formal difference between this situation and the previous ones in this paper: the recommendation notion for Spring does not always map closed sets to closed sets. For example, it maps the closed set $[0,1]$ to the non-closed set $(0,1/2] \cup \{1\}$; that is, the collection of credences which are evaluated as acceptable according to some member of $[0,1]$ is $(0,1/2] \cup \{1\}$. Whenever the recommendation notion is given by a closed map, there is a credal set compatible with Recommendation and Acceptability Constraint.[16] But some recommendation notions, such as that of Spring, do not give closed maps.

One might at this point simply admit that some epistemic dilemmas remain, but nonetheless acknowledge that moving to consider Recommendation and Acceptability Constraint has allowed us to avoid many natural ones. However, I would like to propose an alternative answer which allow us to always avoid epistemic dilemmas.

One option is to weaken Recommendation Constraint to only apply to closed properties. This is equivalent to what I proposed in Campbell-Moore (2021). However, I instead propose an attractive alternative which maintains Recommendation Constraint. For this, we will revisit the notion of indeterminacy

[15] A more general argument runs as follows: Since every credence whatsoever recommends that one have non-zero credence, to satisfy Recommendation Constraint, we must have $0 \notin \mathbb{B}$. If $0 \notin \mathbb{B}$, then every $x \in \mathbb{B}$ recommends $x/2$, which is $\leqslant \frac{\sup \mathbb{B}}{2}$. Since there must be some member of $\mathbb{B}$ which is $> \frac{\sup \mathbb{B}}{2}$, $\mathbb{B}$ fails to determinately satisfy the determinate requirement to be $\leqslant \frac{\sup \mathbb{B}}{2}$.

[16] This relies on the compactness of the space of credences ($[0,1]$, inclusive) or more generally credence functions (functions from propositions to $[0,1]$). It is a consequence of Campbell-Moore (2021, Theorem 8.7.5). Note that this argument does not work when the underlying space is non-compact, for example $(0,1)$ excluding the end-points.

and how to characterise indeterminate epistemic states. We have been working with one's indeterminate epistemic state being specified by a credal set giving us a set of probabilities. However, I now propose to directly focus on the definite judgements. When we introduced credal sets we first highlighted the idea that some properties are determinate, for example that it may be determinate that your credence be $\geqslant 0.5$ but indeterminate whether it is identical to 0.8 or not. We then said that we should think of your indeterminate credences with a set of credence (probability) functions, all those that are compatible with the determinate properties. We now work directly with the determinate judgements rather than the credal sets. So one's indeterminate epistemic state is given by a collection of sets of credence (probability) functions, $\mathcal{D}$. A property P is in $\mathcal{D}$ if it is a definite judgement.[17]

Recommendation and Acceptability Constraint are stated in a way that means they can directly apply to indeterminate states so conceived. We just need to ask which properties are determinately recommended, or determinately acceptable, and which are determinately satisfied. Note that, P is determinately recommended means "P is recommended" is definite, i.e., that $\{x \mid$ every credence acceptable according to x satisfies $P\} \in \mathcal{D}$.

Recommendation and Acceptability Constraint can then be satisfied in a case like Spring by adopting the following epistemic state, $\mathcal{D}$:

Definitely, one's credence is $\leqslant 1$ and non-zero.	$(0, 1] \in \mathcal{D}$
Definitely, one's credence is $\leqslant 1/2$ and non-zero.	$(0, 1/2] \in \mathcal{D}$
Definitely, one's credence is $\leqslant 1/4$ and non-zero.	$(0, 1/4] \in \mathcal{D}$
Definitely, one's credence is $\leqslant 1/8$ and non-zero.	$(0, 1/8] \in \mathcal{D}$
Etc	

This epistemic state does not correspond to any credal set. In fact, there is no credence whatsoever which has all these properties, that is, $\bigcap \mathcal{D} = \varnothing$. Any finite collection of properties is consistent, it is just when the infinitely many properties are taken together that inconsistency arises. I propose that such epistemic states can nonetheless be rational (see also Campbell-Moore, forthcoming). The structural constraint I impose on such epistemic states being rational is that they should be a proper filter: be closed under finite intersection and supersets, and not contain $\varnothing$.[18]

Once we move to this 'filter' model of indeterminate belief, Recommendation and Acceptability Constraint are always satisfiable and epistemic dilemmas are avoided in full generality. A sketch of this result is given in appendix A. This result relies on our more general picture of indeterminate belief, characterised by definite judgements and where we merely impose the finite intersection property.

6 The Revenge Challenge: The Archer Case

Recommendation and Acceptability Constraint are always satisfiable (once the model of indeterminate belief is relaxed as discussed in section 5). So such epistemic dilemmas are always avoided.

[17]Formally, this is related to the move made by Moss (2018) where we conceive of you as having beliefs with probabilistic contents rather than a credence function, or an imprecise credence given by a set of probability functions. In both settings, we consider your epistemic state given by a collection of sets of probability functions.

[18]It should also be non-empty. See Campbell-Moore (forthcoming) for further details.

A reader may wonder about the following kind of case, which is very similar to Ring Toss except it explicitly specifies that if you have indeterminate credences, you will miss.

Archer

> You are an archer in the process of taking your shot. If your credence that you'll hit the bullseye is determinately $\geqslant 0.5$, then you'll be tense and miss it. And if not, then you'll hit it. And you know this about yourself.
> *Joyce, 2018.*

This is the case considered in Joyce (2018). It parallels the definite-liar, which is offered as a revenge challenge for this account of the liar paradox. There is a significant amount of work on revenge challenges for the liar paradox, and similar analysis would need to be done for this approach to epistemic dilemmas.

This case seems to cause a problem because being indeterminate between 0 and 1 undermines its own adoption. In the Archer case, if you are indeterminate between 0 and 1, then your credence is not determinately $\geqslant 0.5$, and thus you will definitely hit it. So being indeterminate between 0 and 1 seems to undermine its own adoption.

However, the rationality constraints on the indeterminate that we spelled out were governed by Recommendation and Acceptability Constraint. These constraints are given in a supervaluational way: what is required of the indeterminate believer supervenes on the recommendation notion governing the members of the credal set. So the proposed way of reasoning that $\{0, 1\}$ undermines its own adoption does not following Recommendation and Acceptability Constraint, but instead is directly encoded by the situation. To properly account for this case, we should spell out recommendation notion which also considers the indeterminacy in your attitudes. Perhaps we then can satisfy Recommendation and Acceptability Constraint by it being indeterminate whether you determinately have credence 0 or determinately have credence 1.

The paradox only reappears when we assume that your indeterminate attitude is determinately held. That when your credence is not determinately $\geqslant 0.5$ it is determinate that your credence is not determinately $\geqslant 0.5$ (cf. McGee, 1989, p. 538). This is something that we have to reject.

Further investigation is required into such revenge challenges. What it highlights is that it not really the move to indeterminate credences that was important, but instead the move from looking at recommendations to determinate recommendations, that is, our constraints Recommendation and Acceptability Constraint.

7 Conclusion

We have proposed an account of rationality that applies to one's indeterminate credal state. Rationality and our picture of indeterminate credences is supervaluational in spirit, with rationality constraints on the indeterminate believers are guided by 'recommendation' which governs the credal members. It may be indeterminate what is recommended, but one should determinately satisfy any determinate recommendations. The two rationality constraints on the indeterminate attitudes we have given are:

Recommendation Constraint
Every determinate recommendation should be determinately satisfied.

Acceptability Constraint
Every determinately satisfied property should be determinately acceptable.

Our account is inspired by, and is part of a general account of, undermining or self-referential notions, such as that of truth and the liar paradox.

Even when what is recommended of you depends on what credences you adopt in an undermining way, there are always indeterminate attitudes that can be rationally adopted, following Recommendation and Acceptability Constraint. (Though they possibly have to be given by definite judgements which do not give any credal set.) Thus the seeming epistemic dilemmas are avoided.

We obtain an attractive principles of rationality. For example, if recommendation is governed by "follow the evidence", then we obtain principles on one's indeterminate epistemic state that say one should determinately follow any determinate evidence. In general, the principles that we obtain are narrow scope principles. We suggest that these narrow scope principles encode the important content of the desirable principles and they allow epistemic dilemmas to be avoided.

In conclusion, Recommendation and Acceptability Constraint offers an attractive picture of rationality which avoids epistemic dilemmas that seem to arise when one's adopted credences affect what is epistemically acceptable in an undermining way.

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A Result that Epistemic Dilemmas are Avoided

Theorem A.1. *A recommendation notion is given by a function, $A : \mathsf{Creds} \to \wp(\mathsf{Creds}) \setminus \varnothing$. For any recommendation notion, there is some proper filter, $\mathcal{D}$, i.e.:*

- $\mathcal{D} \neq \varnothing$ *and* $\varnothing \notin \mathcal{D}$.
- *If* $P, Q \in \mathcal{D}$ *then* $P \cap Q \in \mathcal{D}$.
- *If* $P \in \mathcal{D}$ *and* $Q \supseteq P$ *then* $Q \in \mathcal{D}$.

Where:

- *Recommendation Constraint:* $\{x \mid A(x) \subseteq P\} \in \mathcal{D} \implies P \in \mathcal{D}$.
- *Acceptability Constraint:* $P \in \mathcal{D} \implies \{x \mid A(x) \cap P \neq \varnothing\} \in \mathcal{D}$.

Proof sketch. Construct a transfinite sequence:

- $\mathcal{D}_0 = \{\mathsf{Creds}\}$;
- $P \in \mathcal{D}_{\alpha+1}$ iff $\{x \mid A(x) \subseteq P\} \in \mathcal{D}_\alpha$.
- $\mathcal{D}_\lambda = \bigcup_{\alpha<\lambda} \mathcal{D}_\alpha$ for λ a limit ordinal.

One then shows that for $\alpha < \beta$, $\mathcal{D}_\alpha \subseteq \mathcal{D}_\beta$ by showing that the relevant jump is monotone; and can then conclude that this will reach a fixed-point, some $\mathcal{D}_\sigma = \mathcal{D}_{\sigma+1}$, which will then satisfy Recommendation and Acceptability Constraint.

One should then prove that it is a proper filter by transfinite induction. For the successor stage, this requires that every credence evaluates some credence as acceptable. For the limit stage it is important that the notion of a filter only requires the finite intersection property. $\square$