Probabilistic uncertainty and non-classical logics

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Model of belief and uncertainty

A nice picture of our uncertainty:

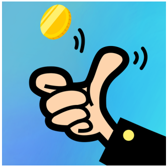
We attach numerical 'degrees of belief' to various outcomes (sentences).

- Will the coin land Heads?

  0.5

- Will the train be on time?

  0.75



These are numbers between 0 and 1.

Rationality? The axioms of probability, e.g. $\mathrm{b}(\varphi \vee \neg\varphi) = 1$; $\mathrm{b}(\varphi) + \mathrm{b}(\neg\varphi) = 1$. Accuracy-argument. Dutch book argument.

Non-classicality

- Liar: Liar is not true

Our account of Liar might involve non-classical logics.
Perhaps Liar is neither True nor False. (I'm esp. thinking Strong Kleene)

Degree of belief 0? or 0.5?

- Is this a heap?

Perhaps it's indeterminate; various precisifications of heap.

Degree of belief 0? or 0.5?

Non-classical probabilism

A suggestion in the literature, Williams and Paris:

We choose a particular number as the 'cognitive load' for these non-classical statuses.

Non-classical probabilism: One's degrees of belief function should be a "weighted average" (in the convex hull) of the cognitive loads.

We can give accuracy arguments (Williams) and Dutch book arguments (Paris) for this non-classical probabilism.

But. . .

Non-classical opinions

Perhaps we need new options
for the kinds of attitudes we can adopt.

Don't assign individual numerical degrees of belief.

If truth values can be indeterminate, perhaps it can be indeterminate whether I believe it to degree 0.5 or not.

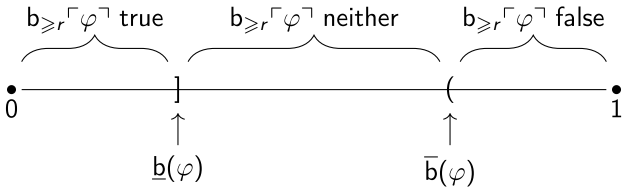
Non-classical models of belief – Three valued logics

Liar is neither True nor False.

'My degree of belief in Liar is $\geqslant 0.5$' – neither True nor False.

How to represent these opinions? With **two numbers**.

$$\underline{\mathrm{b}}(\varphi) \text{ and } \overline{\mathrm{b}}(\varphi)$$



Same model will work for four valued logics, where it can be both.

Non-classical models of belief – Supervaluational logic

- Is this a heap?

Perhaps it's indeterminate; various precisifications of heap, each gives a classical cut-off.

It is indeterminate what my degree of belief is.

Takes one's attitude to be given by

a set of credence functions.

Imprecise credence

Outline

Outline

Self-referential beliefs

PrLiar: I do not have degree of belief $\geqslant 1/2$ in PrLiar

- Adopt $0 \rightsquigarrow \mathrm{PrLiar}$ true $\rightsquigarrow$ deg bel 1.
- Adopt $1 \rightsquigarrow \mathrm{PrLiar}$ false $\rightsquigarrow$ deg bel 0.

Any particular degree of belief you adopt undermines itself.

Does non-classicality help? If we have precise numerical beliefs, then PrLiar seems to get a classical truth value.
And underminingness reappears.

If we allow a non-classical option for belief, like $\underline{\mathrm{b}} = 0$ and $\overline{\mathrm{b}} = 1$, then PrLiar is neither True nor False;
and no underminingness.

Outline

Working with the new models of belief

- Three-valued logics: you have a pair, $\underline{b}$ and $\overline{b}$.
  What values should it take?
- Supervaluational logic: you have a set of credence functions.
  What should your set be like?

How to understand these? Various strategies:

- Try to characterise sentential probabilities by assuming a space of possibilities with a (probabilistic) measure of uncertainty over those.
- Directly think about how the rationality arguments can apply to these frameworks.
  - Accuracy
  - Dutch book.

Numerical non-classical probabilities

There's various ways the world might be like.

	true	false	neither
Omniscient (=ideal) credences:	1	0	? - **0.5**/0/1

TruthTeller could be true, false, or neither — equally likely.

$$b(\text{TruthTeller}) = {}^1\!/\!_3 \times 1 + {}^1\!/\!_3 \times 0 + {}^1\!/\!_3 \times 0.5 = 0.5$$

What it takes to be rational?

There has to be some weights over the strong-Kleene possibilities with $b(\varphi)$ the weighted average of the ideal degrees of belief.

We can also axiomatise these, e.g.:

- If $\varphi \vDash_{\mathsf{SSK}} \psi$, then $\mathrm{b}(\varphi) \leqslant \mathrm{b}(\psi)$.

Their rationality arguments

	true	false	neither
Omniscient credences:	1	0	0.5/0/1

Accuracy argument (Williams):
There should not be a b' which is closer to the omniscient credences, whatever the world is like.

DB argument (Paris):
Consider a bet that pays out the omniscient credence value for φ in each world. Assume you're willing to buy/sell it for £$b(\varphi)$.
You shouldn't take a collection of bets that lead to a loss whatever the world is like.

These both get the same rationality constraint:
weighted averages of the omniscient credences.

But they **assume** (rather than justify) precise credences.

Outline

What should $\underline{b}(\varphi)$ and $\overline{b}(\varphi)$ be like? - First suggestion

Liar is neither. Ideally:

- $\underline{b}(\text{Liar}) = 0$. Degree of belief that Liar is true (or both).
- $\overline{b}(\text{Liar}) = 1$. Degree of belief that Liar is true or neither.

Perhaps we think of it like completely holding my hands up and suspending all judgement. Though that doesn't quite fit when four values.

	true	false	neither
Omniscient lower credence ($\underline{v}$):	1	0	0
Omniscient upper credence ($\overline{v}$):	1	0	1

Of course, I'm not omniscient, so we take averages.
TruthTeller could be true, false, or neither. Suppose equally likely.

$$\underline{b}(\text{TruthTeller}) = {}^1\!/_3 \times 1 + {}^1\!/_3 \times 0 + {}^1\!/_3 \times 0 = {}^1\!/_3$$

$$\overline{b}(\text{TruthTeller}) = {}^1\!/_3 \times 1 + {}^1\!/_3 \times 0 + {}^1\!/_3 \times 1 = {}^2\!/_3$$

Rationality argument for this picture

Accuracy argument: We adopt a pair: $\underline{\mathrm{b}}(\varphi), \overline{\mathrm{b}}(\varphi)$.
Accuracy is closeness of $\underline{\mathrm{b}}$ to $\underline{v}$ and $\overline{\mathrm{b}}$ to $\overline{v}$.

	true	false	neither
$\underline{v}_w(\varphi)$	1	0	0
$\overline{v}_w(\varphi)$	1	0	1

Dutch book argument:

- You will buy/sell a bet that pays out £$\underline{v}_w(\varphi)$ in w for £$\underline{\mathrm{b}}(\varphi)$.
- You will buy/sell a bet that pays out £$\overline{v}_w(\varphi)$ in w for £$\overline{\mathrm{b}}(\varphi)$.

Then there has to be some weights over the possibilities with

- $\underline{\mathrm{b}}(\varphi)$ the weighted average of the omniscient lower credences,
- $\overline{\mathrm{b}}(\varphi)$ the weighted average of the omniscient upper credences.

Axioms

When we take the 'ways the world might be like' to be all strong-Kleene possibilities, we can give axioms:

- $0 \leqslant \underline{b}(\varphi) \leqslant 1$
- If $\varphi \vDash_{K_3} \psi$, then $\underline{b}(\varphi) \leqslant \underline{b}(\psi)$,
- $\underline{b}(\varphi) + \underline{b}(\psi) = \underline{b}(\varphi \vee \psi) + \underline{b}(\varphi \wedge \psi)$
- $\overline{b}(\varphi) = 1 - \underline{b}(\neg\varphi)$.

An alternative suggestion

Dutch book argument:
Consider a bet:

	true	false	neither
Bet pays out:	1	0	Investment is returned

$\underline{\mathrm{b}}(\varphi)$ is my buying price. $\overline{\mathrm{b}}(\varphi)$ is my selling price.

What rationality constraint does this lead to? Not sure, but:

- If φ is definitely neither, anything is allowed, e.g. $0.2, 0.3$.
- If φ is definitely classical, we can still have $\underline{\mathrm{b}}(\varphi) \neq \overline{\mathrm{b}}(\varphi)$.

Accuracy argument: What's analogous? ...

Outline

How to approach the question

We asked: if φ is true / false / indeterminate at w, what's the ideal opinion.

For supervaluationism, this looses some important structure.

Precisifications act 'globally'.

- Cross-sentences (rejecting compositionality):
  Ted and Joe are vaguely bald. But Ted is balder than Joe.
  A set of probability functions, rather than a set-valued fn.
- Cross-worldly: Katie bought a vaguely-red toy car or plane.
  A precisification that makes it red should treat the car and plane world the same.

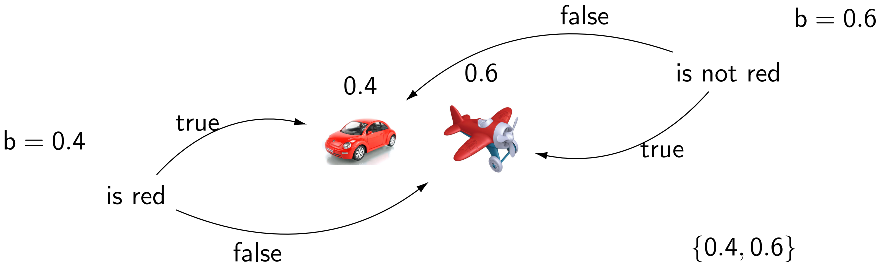
So our structure of possibilities is not just a collection of non-classical worlds.

Rough idea: do all your precise/classical stuff first, and only at the end take sets.

A suggestion for what our sv credences should be

- We have a space of possibilities.
- And a probability measure over them.
- A precisification gives us classical truth values in each world.
- From which we can read off a classical probability function.
- Then one's imprecise probability is the resultant set.

Katie bought a car and it's red; or a plane and not red?



Accuracy?

Think of the different precisifications as a collection of individuals.

Each precisification has an opinion about which precise credence functions are epistemically good or bad. If it's dominated, it's bad.

The collection comes together to discuss the epistemic goodness of various sets. A set of credences is OK if:

- If no-one thinks you're good, you're not allowed in.
- ? Every precisification has someone it thinks is good.

Why? Dunno, but I've thought about this sort of thing just for imprecise creds without supervaluationism and it's powerful.

What does it get us?

Conjecture: It gets us what can be generated from the construction on the previous side, but starting with imprecise probabilities over the worlds.

Analogous DBs?

Outline

Challenges this picture faces - logical uncertainty

Williams and Paris kept the model of belief fixed, just changed what it is to be rational.
For me, the model of belief changes when the logic changes.

What if I'm uncertainty **about logic**.
I don't know if it's SK or SV that's appropriate for Liar.

Should we mix and match models. A set of pairs?
Develop a 'most general' model.

But is this epistemically right? Maybe they're just conflicting.

Summary

There is a nice framework of non-classical probabilities where we keep with precise numerical opinions.

But, perhaps the model of belief should more closely match the model of truth.

- Three-valued logics: Belief as a pair $(\underline{\mathrm{b}}, \overline{\mathrm{b}})$.
- Supervaluational truth: Belief as a set of precise probabilities.

We gave various thoughts on how to understand these non-classical belief systems and give rationality arguments.

Lots of future work!

Thanks!

Summary of what they should be like

Pairs and three-valued:

- We could think about probability of True and not False. Can give accuracy and Dutch book.
- We could be more radical, motivated by Dutch book considerations. What exactly are these like?

Sets and supervaluationism:

- Using a probability over the space of worlds, and each precisification gives us a precise probability.
- Accuracy considerations - think of the precisifications as a group with opinions coming together to judge the imprecise credences.

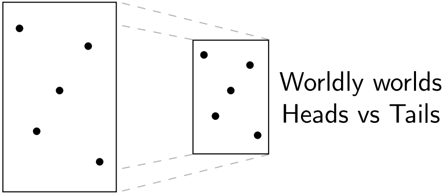
Levels of uncertainty - important for sv

- **Worldly uncertainty.** E.g.
  - A shop sells vaguely-red toy cars and yellow ducks. You're not sure which he's bought.
  - You're not sure how the coin landed. What is b($H \leftrightarrow$ Liar)?
- **Semantic uncertainty.** You don't know which SV model is best for the concept at stake. E.g.
  - You're not sure which precisifications of 'heap' are legitimate.
  - You're not sure which SV fp of Kripke's construction is right.
- (**Logical uncertainty.**) SK or SV

What the possibilities look like for the supervaluational model combining semantic and worldly uncertainty

What are the possibilities?

Each dot is a supervaluational model:
is a set of precisifications
each of which gives truth values at each worldly-worlds



Alternatively, have duplicate worldly-worlds for the semantic uncertainty.
But they should be coordinated somehow, eg when updating beliefs.

Janda's argument for pairs in four valued logics

In four valued logics we have that sentences can be true, false, both or neither.

A difference of truth value should lead to a diff of ideal opinion.

If we stick with numerical credences:

true	false	neither	both	
1	0	0.5	0.5	✗
1	0	0	1	✗
1	0	0.4	0.6	oddly asymmetric ✗

We need new options.